

Bose-Einstein condensation of indirect excitons in coupled quantum wells

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We study the ground-state properties of a quasi-two-dimensional Bose-Einstein condensate of indirect excitons, which are confined in an anisotropic harmonic potential. Incorporating the interactions, we calculate the order parameter variationally. The difficulties in the detection of a Bose-Einstein condensate are also discussed, along with possible ways which would overcome them.

I. INTRODUCTION

Excitons are bound states of electrons and holes which form in excited semiconductors [1], very much like positronium atoms. Since excitons consist of two fermions, at least as long as the mean spacing between them is much larger than their Bohr radius, they are expected to behave as bosons [2]. While the phase transition to a Bose-Einstein condensed phase has been realized experimentally in vapours of alkali-metal atoms [3], the formation of an exciton Bose-Einstein condensate (BEC) has turned out to be more tricky. A lot of experimental effort has been made on the formation of a BEC of excitons in Cu_2O [4,5], and in quantum wells [6]. Excitons resemble atoms, with one major difference being their mass, since they are much lighter particles and as a result they exhibit quantum-mechanical effects more easily. In that respect, excitons are more advantageous than atoms for forming a BEC. On the other hand, their basic disadvantage is their finite lifetime due to various decay mechanisms [7,8], which destroy them.

For a gas of bosons of mass m and density n , the phase transition to a Bose-Einstein

condensed phase occurs when their thermal de-Broglie wavelength becomes comparable to their interparticle distance, which implies that $k_B T_c \sim \hbar^2 n^{2/3}/m$, where T_c is the critical temperature. For T_c being on the order of liquid-helium temperature, i.e., 1 K, and for m being on the order of the electron mass, n turns out to be on the order of 10^{17} cm^{-3} . For such densities, the boson-boson spacing is $\sim 500 \text{ \AA}$, while typically the exciton Bohr radius is smaller than this length scale, and therefore the excitons are expected to behave as point-like (bosonic) particles.

Recently in an interesting paper Butov *et al.* [9] (see also [10]) reported the formation of a degenerate gas of excitons in a quasi-two-dimensional geometry in coupled quantum wells. More precisely, in the experiment of Ref. [9] indirect excitons formed between two parallel layers, after an electric field was applied perpendicular to the two planes. Then, spatially resolving the photoluminescence that comes from the layers, isolated bright localized spots were observed. These spots were attributed to the recombining excitons which have concentrated in regions of local minima (lakes) that exist due to irregularities in the heterostructures. The presence of these local minima is important, since the excitons concentrate in them. Furthermore, the exciton lifetime is long due to the small overlap between the electron and the hole wavefunctions, and also the cooling due to the scattering of excitons with acoustic phonons is enhanced because of the relaxation of the momentum conservation along the direction of confinement. All the above effects favor the formation of a cold and dense gas of excitons, and indeed signs of degenerate behaviour in these lakes were observed.

Motivated by this experiment, we examine in the present paper the ground-state properties of a Bose-Einstein condensate of excitons in such a system [11]. Our goal is to demonstrate some important physics which, in addition to other things, would allow the detection of a Bose-Einstein condensate and not to give a detailed quantitative description. On the other hand, our results should be reliable at least as order-of-magnitude estimates. In that spirit, and since the real system is quite complicated, we make some simplifying assumptions. For example, we neglect the finite lifetime of excitons, since typically this is larger by one or two orders of magnitude [12] than the thermalization timescale [13]. In addition,

the rate of non-radiative, Auger-like processes is expected to be suppressed because of the spatial separation between the electron and the hole of each individual exciton [14].

In what follows we examine in Sec. II the interactions between the excitons and in Sec. III how the statistics due to the Pauli principle between the electrons and/or the holes of different excitons affect them. We then make some estimates in Sec. IV, where we model the in-plane confinement with a harmonic potential. Using a variational ansatz for the order parameter we calculate in Sec. V the energy and the width of the cloud in the case of an asymmetric confining potential. Finally we discuss in Sec. VI how one could get evidence for the existence of a BEC, while Sec. VII summarizes our results. As shown, the width of the condensate is expected to be of the same order as that of the thermal cloud and for finite temperatures, that would obscure the detection of the condensate. In that respect, an important point of our study is that (similarly to the atomic condensates [15]) as long as the trapping potential is *anisotropic*, the kinetic-energy distribution of an expanding BEC (upon release of the excitons from the trap) is also *anisotropic*, while the kinetic-energy distribution of a thermal gas is always *isotropic*. Therefore, any experiment which is sensitive to the anisotropy of the kinetic-energy distribution of the excitons could be used as a “smoking gun” for the detection of a BEC.

II. INTERACTION BETWEEN THE EXCITONS

As mentioned above, the interactions play a very important role in this system. Numerous studies have examined this problem - mostly the effect of the reduced dimensionality and of the band structure of specific materials. It is not the purpose of the present paper to investigate such questions in detail, but rather to follow a method that captures the essential physics we want to demonstrate.

It is important to remember that the electrons and the holes in this system are spatially separated and as a result the corresponding electron-hole pairs (that form in the limit of dilute densities that we consider) have a dipole moment. As shown in Ref. [16] in such

a system the effective exciton-exciton interaction is dominated by the one that reduces to the usual dipole-dipole term for large exciton separations and this is the approach that we follow here. Viewed in another way, this approach is phenomenological. While it is certainly reliable for large separations between the excitons, it is less reliable for short distances, where one needs to worry about the exchange terms between the electrons and the holes of different excitons. However, the dipole moment of the excitons dominates the interaction and our approach is certainly valid for low exciton densities; a similar approach was followed in Ref. [17]. References [18] have studied a problem close to the present one in the context of atomic condensates which are subjected to an external electric field.

More precisely, let us consider the two-exciton problem, i.e., two electrons confined on a plane and two holes confined on a parallel plane, with the planes being separated by a distance D . As in the experiment of Ref. [9] we assume that D is on the order of the Bohr radius a_B of a single exciton (this is a reasonable thing to do, since if $D \sim a_B$ the electron-hole bound state is essentially a p -like hydrogenic state). Fixing the position of, say, the two electrons at an infinite distance, in the lowest state each hole binds with one electron and two excitons form, with the total energy of the system being twice the binding energy of a single exciton. If the two excitons are brought to a finite distance from each other, the interaction between them is given by

$$V(\mathbf{r} - \mathbf{r}') = \frac{e^2}{\epsilon} \left(\frac{2}{|\mathbf{r} - \mathbf{r}'|} - \frac{1}{|\mathbf{r} - \mathbf{r}' + \mathbf{D}|} - \frac{1}{|\mathbf{r} - \mathbf{r}' - \mathbf{D}|} \right), \quad (1)$$

where ϵ is the dielectric constant, and $\mathbf{r} - \mathbf{r}'$ is the vector connecting the electrons or the holes of the two excitons. In our study we assume for simplicity that the exciton dipole moment is parallel to the electric field (i.e., perpendicular to the x - y plane) and we neglect the fluctuations around this orientation. For small distances between the two excitons, $|\mathbf{r} - \mathbf{r}'| \ll D$, Eq. (1) reduces to the usual Coulom interaction between the two electrons and the two holes, $V(\mathbf{r} - \mathbf{r}') = 2e^2/(\epsilon|\mathbf{r} - \mathbf{r}'|)$. For large distances between the two excitons, $|\mathbf{r} - \mathbf{r}'| \gg D$, Eq. (1) reduces to the usual dipole-dipole interaction, $V(\mathbf{r} - \mathbf{r}') = e^2 D^2 (1 - 3 \cos^2 \theta) / \epsilon |\mathbf{r} - \mathbf{r}'|^3$, where θ is the angle between \mathbf{D} and $\mathbf{r} - \mathbf{r}'$. In our case, since the system is quasi-two-

dimensional, $\theta \approx \pi/2$. Therefore $V(\mathbf{r} - \mathbf{r}')$ is purely repulsive, which guarantees that there are no instabilities against the formation of e.g., molecules.

III. EFFECT OF THE PAULI PRINCIPLE

When the separation between the excitons becomes of order a_B , one has to worry about the Pauli principle between the electrons and/or the holes. If the spins of either the electrons, or the holes are parallel, the relative wavefunction between the two excitons is highly suppressed for values of $|\mathbf{r} - \mathbf{r}'|$ smaller than a_B (on the other hand, this effect is absent when both the electrons and the holes have antiparallel spins). As a result, the excitons are not allowed to get to distances shorter than $|\mathbf{r} - \mathbf{r}'| \sim a_B$. Therefore, for exciton-exciton separations on the order of a_B , the true many-body wavefunction behaves in a similar way (since for the low densities we consider, the effect of other excitons is negligible) [19]; however our mean-field approach ignores these correlations at such small scales. For this reason, to calculate the interaction energy when the Pauli principle is active, we will use Eq. (1) assuming that the two excitons cannot get to distances closer than a_B . It is important to point out two things concerning these implications that result from the Pauli principle: (i) as shown in detail below, the energy of the system is not very sensitive to the choice of the cutoff length (which we choose to be a_B for simplicity), and (ii) the bosonic nature of the electron-hole pairs is not affected, as long as the mean exciton-exciton spacing is larger than a_B , which is indeed the case in the experiment of Ref. [9].

IV. ESTIMATES

Before we calculate the order parameter of the condensate, it is instructive to make some estimates. Let us assume that the excitons are confined on the x - y plane within a lake of radius d . For low enough densities the dipole expansion can be used and in this case the energy per particle of a BEC of excitons is (apart from numerical factors of order unity),

$$\mathcal{E}(d) \sim \frac{\hbar^2}{2md^2} + \frac{1}{2}m\omega^2 d^2 + \frac{Ne^2 D^2}{\epsilon d^2 D}, \quad (2)$$

where N is the number of excitons. In the above equation we have assumed that the excitons are trapped in a harmonic potential of frequency ω (corresponding to the confinement in the lake). Equation (2) can also be written as

$$\mathcal{E}(d) \sim \frac{\hbar^2}{2md^2} \left(1 + \frac{ND}{a_B}\right) + \frac{1}{2}m\omega^2 d^2, \quad (3)$$

where $a_B = \epsilon \hbar^2 / \mu_x e^2$ is the exciton Bohr radius, with $\mu_x = m_e m_h / m$ being the reduced mass of the effective electron mass m_e and the effective hole mass m_h , and $m = m_e + m_h$. For $\mu_x \sim 0.1m_0$, where m_0 is the electron mass and $\epsilon \sim 10$, then $a_B \sim 100 \text{ \AA}$, which is $\sim D$. Under typical conditions, $N \sim 10^6$, the first term on the right side of Eq. (3) is negligible. Under this assumption, the confinement due to the lake is balanced by the (repulsive) interactions between the dipoles, while the kinetic energy (due to the Heisenberg uncertainty principle) is negligible, very much like the Thomas-Fermi approximation in atomic condensates [20],

$$\mathcal{E}(d) \sim \frac{\hbar^2}{2md^2} \frac{ND}{a_B} + \frac{1}{2}m\omega^2 d^2, \quad (4)$$

and thus the radius of the condensate is

$$d_0/a_{\text{osc}} = c_1 (ND/a_B)^{1/4}, \quad (5)$$

with $a_{\text{osc}} = (\hbar/m\omega)^{1/2}$ being the oscillator length and c_1 being a constant of order unity. Also,

$$\mathcal{E}(d_0)/\hbar\omega = c_2 (ND/a_B)^{1/2}, \quad (6)$$

where c_2 is a constant of order unity. A useful formula is the one that compares the radius of the condensate d_0 with the radius of the thermal cloud R . Since $m\omega^2 R^2 \sim k_B T$, and assuming that $k_B T_c \sim \hbar\omega N^{1/2}$ [3],

$$\frac{d_0}{R} \sim \left(\frac{\hbar\omega}{k_B T}\right)^{1/2} \left(\frac{ND}{a_B}\right)^{1/4} \sim \left(\frac{T_c}{T}\right)^{1/2} \left(\frac{D}{a_B}\right)^{1/4}. \quad (7)$$

Remarkably, the ratio d_0/R does not depend on the number of excitons N . Turning to the speed of sound c , $mc^2 \sim \langle V \rangle$, where $\langle V \rangle \sim Ne^2D/\epsilon d_0^2$ is the interaction energy per particle, and thus $c \sim (\hbar/md_0)(ND/a_B)^{1/2}$. Also, the coherence or healing length ξ , satisfies the equation $\hbar^2/2m\xi \sim \langle V \rangle$, and thus $\xi/d_0 \sim (a_B/ND)^{1/2}$. Finally the critical frequency Ω_c for creating a vortex state is $\sim (\hbar/md_0^2) \ln(d_0/\xi)$.

Let us make some numerical estimates now. Since the radius R of a thermal cloud of excitons in the lakes is $\approx 10 \mu\text{m}$ for $T \approx 1 \text{ K}$ [9], the formula $m\omega^2 R^2 \sim k_B T$ implies that $\omega \sim 10^9 \text{ Hz}$ for m being on the order of $0.1m_0$. Given this value for ω , the oscillator length a_{osc} is $\sim 1 \mu\text{m}$, and $\hbar\omega \sim 5 \times 10^{-4} \text{ meV}$. Therefore, since $D \sim a_B$, then $ND/a_B \sim N \sim 10^6$, and the size of the condensate d_0 is $\sim 30 \mu\text{m}$, the surface density n_{2D} is $\sim 3 \times 10^{10} \text{ cm}^{-2}$ and the mean exciton-exciton spacing $\sim d_0/N^{1/2}$ is $\sim 300 \text{ \AA}$. The fact that $n_{2D} a_B^2$ is ~ 0.03 , i.e., much less than unity, where n_{2D} is the surface density, justifies the treatment of the excitons as bosons and the use of Eq. (1) as the appropriate expression for the interaction between the excitons. Turning to the energy per particle of the condensate, this is $\sim 1 \text{ meV}$. The transition temperature T_c is $\sim 5 \text{ K}$, while for $T = T_c$, Eq. (7) implies that $d_0 \approx R$. Also the speed of sound c is $\sim 10^6 \text{ cm/s}$, and the coherence length ξ is $\sim 100 \text{ \AA}$, which gives the typical size of vortices [10]. Since $\xi/d_0 \sim 10^{-3}$, the superfluid properties of this system should resemble those of helium (and not those of nuclei). Finally the critical frequency Ω_c for creating a vortex state is $\sim 5 \times 10^5 \text{ Hz}$.

V. VARIATIONAL CALCULATION OF THE ORDER PARAMETER

Let us now get more precise. If $\Psi(\mathbf{r})$ is the order parameter of a BEC of excitons, as long as the estimates made earlier are accurate, it is enough to consider only the relevant terms in the Hamiltonian,

$$\left(V_t(\mathbf{r}) + \int V(\mathbf{r} - \mathbf{r}') |\Psi(\mathbf{r}')|^2 d^2 r' \right) \Psi(\mathbf{r}) = \mu \Psi(\mathbf{r}), \quad (8)$$

where $V_t(\mathbf{r})$ is the trapping potential and μ is the chemical potential. We will now use the variational approach to calculate the order parameter and the energy. For simplicity we

restrict ourselves to two dimensions [21], i.e., on the x - y plane and assume that the trapping potential has the form

$$V_t(\mathbf{r}) = \frac{1}{2}m\omega^2(x^2 + \lambda^2 y^2), \quad (9)$$

with $\omega = \omega_x$, and $\lambda = \omega_y/\omega_x$, and also

$$\Psi(\mathbf{r}) = \frac{N^{1/2}}{\pi^{1/2}(a_x a_y)^{1/2}} \exp(-x^2/2a_x^2 - y^2/2a_y^2), \quad (10)$$

where a_x and a_y are variational parameters. Then, the energy per particle is,

$$\mathcal{E} = \frac{1}{N} \left(\int V_t(\mathbf{r}) |\Psi(\mathbf{r})|^2 d^2 r + \frac{1}{2} \int V(\mathbf{r} - \mathbf{r}') |\Psi(\mathbf{r})|^2 |\Psi(\mathbf{r}')|^2 d^2 r d^2 r' \right). \quad (11)$$

Starting with the expectation value of V_t , $\langle \Psi | V_t | \Psi \rangle / N = m\omega^2(a_x^2 + \lambda^2 a_y^2)/4$. Turning to the interaction energy

$$\langle V \rangle = \frac{e^2}{2N\epsilon} \int |\Psi(\mathbf{r})|^2 |\Psi(\mathbf{r}')|^2 \left(\frac{2}{|\mathbf{r} - \mathbf{r}'|} - \frac{1}{|\mathbf{r} - \mathbf{r}' + \mathbf{D}|} - \frac{1}{|\mathbf{r} - \mathbf{r}' - \mathbf{D}|} \right) d^2 r d^2 r', \quad (12)$$

where, as we argued earlier, $|\mathbf{r} - \mathbf{r}'| \geq a_B$. To evaluate the above integral, we use the relative coordinates $\mathbf{r}_{12} = \mathbf{r} - \mathbf{r}'$, and $\mathbf{R}_{12} = (\mathbf{r} + \mathbf{r}')/2$. Equation (12) then involves an integration over \mathbf{R}_{12} and \mathbf{r}_{12} ,

$$\langle V \rangle = \frac{Ne^2}{\pi^2 \epsilon (a_x a_y)^2} \int e^{-2R_{12}^2 f(\phi)} d^2 R_{12} \int \frac{1}{r_{12}} \left(1 - \frac{r_{12}}{\sqrt{r_{12}^2 + D^2}} \right) e^{-r_{12}^2 f(\phi')/2} d^2 r_{12}, \quad (13)$$

where $f(\phi) = (\cos \phi/a_x)^2 + (\sin \phi/a_y)^2$. Since $D \ll a_x, a_y$, we take the exponential in the integral over \mathbf{r}_{12} to be equal to unity, and thus

$$\begin{aligned} \int_{a_B}^{\infty} \frac{1}{r_{12}} \left(1 - \frac{r_{12}}{\sqrt{r_{12}^2 + D^2}} \right) e^{-r_{12}^2 f(\phi')/2} d^2 r_{12} &\approx \int_{a_B}^{\infty} \frac{1}{r_{12}} \left(1 - \frac{r_{12}}{\sqrt{r_{12}^2 + D^2}} \right) d^2 r_{12} \\ &= 2\pi(\sqrt{a_B^2 + D^2} - a_B). \end{aligned} \quad (14)$$

Combining the equations above, the energy per particle is

$$\mathcal{E} = \frac{1}{4}m\omega^2(a_x^2 + \lambda^2 a_y^2) + \frac{2Ne^2(\sqrt{a_B^2 + D^2} - a_B)}{\pi\epsilon(a_x a_y)^2} \int e^{-2R_{12}^2 f(\phi)} d^2 R_{12} \quad (15)$$

expressed in terms of the two variational parameters a_x, a_y , and λ . For $\lambda = 1$, then $a_x = a_y = d$, $f = 1/d^2$, the integral in Eq. (15) is equal to $\pi d^2/2$, and Eq. (15) gets the simple form

$$\mathcal{E} = \frac{1}{2}m\omega^2 d^2 + \frac{Ne^2(\sqrt{a_B^2 + D^2} - a_B)}{\epsilon d^2}. \quad (16)$$

As Eq. (16) implies, the energy of the system scales as $(\sqrt{a_B^2 + D^2} - a_B)^{1/2}$, which shows that indeed \mathcal{E} is not very sensitive to the cutoff length for $a_B \sim D$. Furthermore, if the cutoff length in Eq. (14) is set equal to zero,

$$\mathcal{E} = \frac{1}{2}m\omega^2 d^2 + \frac{Ne^2 D}{\epsilon d^2}, \quad (17)$$

in agreement with the estimates of Eq. (4). Equation (17) implies that c_1 in Eq. (5) is equal to $(2m/\mu_x)^{1/4}$, and c_2 in Eq. (6) is equal to $(2m/\mu_x)^{1/2}$, where $m = m_e + m_h$.

Returning to the more general problem of $\lambda \neq 1$, we have minimized the energy of Eq. (15), setting for simplicity the cutoff length equal to zero. Measuring a_x and a_y in the second term of this expression in units of a_{osc} , the energy scale $2Ne^2 D/\pi\epsilon a_{\text{osc}}^2$ can be written as $\hbar\omega(2ND/\pi a_B)(m/\mu_x)$. We choose $(2ND/\pi a_B)(m/\mu_x)$ to be equal to 0.5×10^6 . We also consider the value $\lambda = 0.8$, which corresponds to a trapping potential with equipotential lines which are elongated along the y axis, with a ratio of the oscillator lengths $a_{\text{osc},x}/a_{\text{osc},y}$ equal to $\lambda^{1/2} \approx 0.894$. Minimizing numerically the energy, we find that $\mathcal{E}/\hbar\omega \approx 1.119 \times 10^3$ with $a_x/a_{\text{osc}} \approx 33.3$ and $a_y/a_{\text{osc}} \approx 42.0$. It is interesting that the ratio $a_x/a_y \approx 0.793$ is smaller than $\lambda^{1/2}$. Therefore, the interactions enhance the anisotropy of the cloud due to the trapping potential, (i.e., they make the interacting cloud more elongated along the y axis), as compared to a non-interacting gas, which would result from the uncertainty principle. A similar effect has been observed in atomic condensates with contact interactions [15,20].

VI. DISCUSSION OF THE RESULTS AND EXPERIMENTAL RELEVANCE

In the last part of this study we examine how the above results can be used in practice. Getting indisputable evidence and clear signatures for the existence of a BEC of excitons is

not at all a trivial problem [23], since even for small but finite temperatures the condensate co-exists with the thermal cloud. More precisely, as Eq. (7) implies, the width of the condensate is, even for $T = T_c$, comparable or even larger than that of the thermal component, and this ratio gets even larger as the temperature decreases. This is opposite to the atomic condensates, where typically the thermal cloud is much broader than the condensate. This fact would certainly obscure the detection of a BEC of excitons in the present system.

In the alkali-metal atoms, in the first experiment where the formation of a BEC was reported [15], the atoms were confined in a disk-shaped trap. Following their release from the trap, for very low – essentially zero – temperatures, the kinetic-energy distribution of the atoms was observed to be anisotropic, while for higher temperatures (but still lower than T_c) a more broad isotropic background was also observed, corresponding to the thermal component. Above T_c , only the isotropic component remained. Therefore, the anisotropic expansion of the cloud is a *unique* characteristic of a BEC, and *does not* show up in a thermal cloud.

Turning to the present problem, ideally one could think about releasing somehow the excitons from the lake and watching them as they expand. Actually, such a possibility has already been studied theoretically [24]. In such an experiment, any sign of anisotropic expansion of the cloud on the x - y plane below some temperature (the transition temperature T_c) would provide strong evidence in favour of the presence of a BEC of excitons. On the other hand, from the estimates made earlier, one can see that the typical velocity of expansion is expected to be $\sim (\mathcal{E}/m)^{1/2} \sim 10^6$ cm/s. Since excitons with velocities up to $\approx 1.4 \times 10^6$ are radiatively active [25], the expanding excitons may or may not be radiatively active. Additionally, since the velocity of expansion is close to the sound velocity of the material ($\approx 3.7 \times 10^5$ cm/s), the interaction with phonons will slow them down. Apart from these possible difficulties, more generally one should remember that any experiment that is sensitive to the directionality of the kinetic-energy distribution of the excitons could be used to provide evidence for the formation of a BEC.

VII. SUMMARY

To summarize, in this study we examined the ground-state properties of a quasi-two-dimensional Bose-Einstein condensate of indirect excitons, which are confined in an anisotropic harmonic potential. Incorporating the interactions we made estimates for the size of the cloud, the energy per particle, the density, the speed of sound, the coherence length, and the critical frequency for creating a vortex state. We also calculated the order parameter variationally within the mean-field approximation and we examined the effect of the Pauli principle. Concerning the detection of a BEC, since the size of the condensate is expected to be of the same order as that of the thermal cloud, we examined the possibility of releasing the excitons from an anisotropic trap, which would result in an anisotropic kinetic-energy distribution and it would provide evidence for the presence of a BEC. An interesting result is that the interacting gas exhibits this anisotropy in a more pronounced way as compared to an ideal gas.

VIII. ACKNOWLEDGMENTS

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